R.5 Exercises

Find the domain of each rational expression. ***See Example 1.***

1. $\dfrac{x+3}{x-6}$
2. $\dfrac{2x-4}{x+7}$
3. $\dfrac{3x+7}{(4x+2)(x-1)}$
4. $\dfrac{9x+12}{(2x+3)(x-5)}$
5. $\dfrac{12}{x^2+5x+6}$
6. $\dfrac{3}{x^2-5x-6}$
7. $\dfrac{x^2-1}{x+1}$
8. $\dfrac{x^2-25}{x-5}$
9. $\dfrac{x^3-1}{x-1}$
10. *Concept Check* Use specific values for x and y to show that in general, $\frac{1}{x}+\frac{1}{y}$ is not equivalent to $\frac{1}{x+y}$.

Write each rational expression in lowest terms. ***See Example 2.***

11. $\dfrac{8x^2+16x}{4x^2}$
12. $\dfrac{36y^2+72y}{9y^2}$
13. $\dfrac{3(3-t)}{(t+5)(t-3)}$
14. $\dfrac{-8(4-y)}{(y+2)(y-4)}$
15. $\dfrac{8k+16}{9k+18}$
16. $\dfrac{20r+10}{30r+15}$
17. $\dfrac{m^2-4m+4}{m^2+m-6}$
18. $\dfrac{r^2-r-6}{r^2+r-12}$
19. $\dfrac{8m^2+6m-9}{16m^2-9}$
20. $\dfrac{6y^2+11y+4}{3y^2+7y+4}$
21. $\dfrac{x^3+64}{x+4}$
22. $\dfrac{y^3-27}{y-3}$

Find each product or quotient. ***See Example 3.***

23. $\dfrac{15p^3}{9p^2} \div \dfrac{6p}{10p^2}$
24. $\dfrac{8r^3}{6r} \div \dfrac{5r^2}{9r^3}$
25. $\dfrac{2k+8}{6} \div \dfrac{3k+12}{2}$
26. $\dfrac{5m+25}{10} \div \dfrac{6m+30}{12}$
27. $\dfrac{x^2+x}{5} \cdot \dfrac{25}{xy+y}$
28. $\dfrac{y^3+y^2}{7} \cdot \dfrac{49}{y^4+y^3}$
29. $\dfrac{4a+12}{2a-10} \div \dfrac{a^2-9}{a^2-a-20}$
30. $\dfrac{6r-18}{9r^2+6r-24} \div \dfrac{4r-12}{12r-16}$
31. $\dfrac{p^2-p-12}{p^2-2p-15} \cdot \dfrac{p^2-9p+20}{p^2-8p+16}$
32. $\dfrac{x^2+2x-15}{x^2+11x+30} \cdot \dfrac{x^2+2x-24}{x^2-8x+15}$
33. $\dfrac{m^2+3m+2}{m^2+5m+4} \div \dfrac{m^2+5m+6}{m^2+10m+24}$
34. $\dfrac{y^2+y-2}{y^2+3y-4} \div \dfrac{y^2+3y+2}{y^2+4y+3}$
35. $\dfrac{x^3+y^3}{x^3-y^3} \cdot \dfrac{x^2-y^2}{x^2+2xy+y^2}$
36. $\dfrac{x^2-y^2}{(x-y)^2} \cdot \dfrac{x^2-xy+y^2}{x^2-2xy+y^2} \div \dfrac{x^3+y^3}{(x-y)^4}$
37. $\dfrac{xz-xw+2yz-2yw}{z^2-w^2} \cdot \dfrac{4z+4w+xz+wx}{16-x^2}$
38. $\dfrac{ac+ad+bc+bd}{a^2-b^2} \cdot \dfrac{a^3-b^3}{2a^2+2ab+2b^2}$
39. *Concept Check* Which of the following rational expressions is equivalent to -1? In choices A, B, and D, $x \neq -4$, and in choice C, $x \neq 4$. (*Hint:* There may be more than one answer.)

    A. $\dfrac{x-4}{x+4}$ **B.** $\dfrac{-x-4}{x+4}$ **C.** $\dfrac{x-4}{4-x}$ **D.** $\dfrac{x-4}{-x-4}$

40. Explain how to find the least common denominator of several fractions.

Perform each addition or subtraction. ***See Example 4.***

41. $\frac{3}{2k}+\frac{5}{3k}$

42. $\frac{8}{5p}+\frac{3}{4p}$

43. $\frac{1}{6m}+\frac{2}{5m}+\frac{4}{m}$

44. $\frac{8}{3p}+\frac{5}{4p}+\frac{9}{2p}$

45. $\frac{1}{a}-\frac{b}{a^2}$

46. $\frac{3}{z}+\frac{x}{z^2}$

47. $\frac{5}{12x^2y}-\frac{11}{6xy}$

48. $\frac{7}{18a^3b^2}-\frac{2}{9ab}$

49. $\frac{17y+3}{9y+7}-\frac{-10y-18}{9y+7}$

50. $\frac{7x+8}{3x+2}-\frac{x+4}{3x+2}$

51. $\frac{1}{x+z}+\frac{1}{x-z}$

52. $\frac{m+1}{m-1}+\frac{m-1}{m+1}$

53. $\frac{3}{a-2}-\frac{1}{2-a}$

54. $\frac{4}{p-q}-\frac{2}{q-p}$

55. $\frac{x+y}{2x-y}-\frac{2x}{y-2x}$

56. $\frac{m-4}{3m-4}-\frac{5m}{4-3m}$

57. $\frac{4}{x+1}+\frac{1}{x^2-x+1}-\frac{12}{x^3+1}$

58. $\frac{5}{x+2}+\frac{2}{x^2-2x+4}-\frac{60}{x^3+8}$

59. $\frac{3x}{x^2+x-12}-\frac{x}{x^2-16}$

60. $\frac{p}{2p^2-9p-5}-\frac{2p}{6p^2-p-2}$

Simplify each expression. ***See Example 5.***

61. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

62. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

63. $\dfrac{\frac{1}{x+1}-\frac{1}{x}}{\frac{1}{x}}$

64. $\dfrac{\frac{1}{y+3}-\frac{1}{y}}{\frac{1}{y}}$

65. $\dfrac{1+\frac{1}{1-b}}{1-\frac{1}{1+b}}$

66. $\dfrac{2+\frac{2}{1+x}}{2-\frac{2}{1-x}}$

67. $\dfrac{\frac{1}{a^3+b^3}}{\frac{1}{a^2+2ab+b^2}}$

68. $\dfrac{\frac{1}{x^3-y^3}}{\frac{1}{x^2-y^2}}$

69. $\dfrac{m-\frac{1}{m^2-4}}{\frac{1}{m+2}}$

70. $\dfrac{y+\frac{1}{y^2-9}}{\frac{1}{y+3}}$

71. $\dfrac{\frac{3}{p^2-16}+p}{\frac{1}{p-4}}$

72. $\dfrac{\frac{6}{x^2-25}+x}{\frac{1}{x-5}}$

73. $\dfrac{\frac{y+3}{y}-\frac{4}{y-1}}{\frac{y}{y-1}+\frac{1}{y}}$

74. $\dfrac{\frac{x+4}{x}-\frac{3}{x-2}}{\frac{x}{x-2}+\frac{1}{x}}$

75. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

76. $\dfrac{\frac{-2}{x+h}-\frac{-2}{x}}{h}$

77. $\dfrac{\frac{1}{(x+h)^2+9}-\frac{1}{x^2+9}}{h}$

78. $\dfrac{\frac{2}{(x+h)^2+16}-\frac{2}{x^2+16}}{h}$