R.4 Exercises

Factor out the greatest common factor from each polynomial. ***See Examples 1 and 2.***

1. $12m + 60$ **2.** $15r - 27$ **3.** $8k^3 + 24k$

4. $9z^4 + 81z$ **5.** $xy - 5xy^2$ **6.** $5h^2j + hj$

7. $-4p^3q^4 - 2p^2q^5$ **8.** $-3z^5w^2 - 18z^3w^4$

9. $4k^2m^3 + 8k^4m^3 - 12k^2m^4$ **10.** $28r^4s^2 + 7r^3s - 35r^4s^3$

11. $2(a + b) + 4m(a + b)$ **12.** $6x(a + b) - 4y(a + b)$

13. $(5r - 6)(r + 3) - (2r - 1)(r + 3)$ **14.** $(4z - 5)(3z - 2) - (3z - 9)(3z - 2)$

15. $2(m - 1) - 3(m - 1)^2 + 2(m - 1)^3$ **16.** $5(a + 3)^3 - 2(a + 3) + (a + 3)^2$

17. *Concept Check* When directed to completely factor the polynomial $4x^2y^5 - 8xy^3$, a student wrote $2xy^3(2xy^2 - 4)$. When the teacher did not give him full credit, he complained because when his answer is multiplied out, the result is the original polynomial. Give the correct answer.

18. *Concept Check* Kurt factored $16a^2 - 40a - 6a + 15$ by grouping and obtained $(8a - 3)(2a - 5)$. Callie factored the same polynomial and gave an answer of $(3 - 8a)(5 - 2a)$. Which answer is correct?

Factor each polynomial by grouping. ***See Example 2.***

19. $6st + 9t - 10s - 15$ **20.** $10ab - 6b + 35a - 21$

21. $2m^4 + 6 - am^4 - 3a$ **22.** $15 - 5m^2 - 3r^2 + m^2r^2$

23. $p^2q^2 - 10 - 2q^2 + 5p^2$ **24.** $20z^2 - 8x + 5pz^2 - 2px$

Factor each trinomial, if possible. ***See Examples 3 and 4.***

25. $6a^2 - 11a + 4$ **26.** $8h^2 - 2h - 21$ **27.** $3m^2 + 14m + 8$

28. $9y^2 - 18y + 8$ **29.** $15p^2 + 24p + 8$ **30.** $9x^2 + 4x - 2$

31. $12a^3 + 10a^2 - 42a$ **32.** $36x^3 + 18x^2 - 4x$ **33.** $6k^2 + 5kp - 6p^2$

34. $14m^2 + 11mr - 15r^2$ **35.** $5a^2 - 7ab - 6b^2$ **36.** $12s^2 + 11st - 5t^2$

37. $12x^2 - xy - y^2$ **38.** $30a^2 + am - m^2$ **39.** $24a^4 + 10a^3b - 4a^2b^2$

40. $18x^5 + 15x^4z - 75x^3z^2$ **41.** $9m^2 - 12m + 4$ **42.** $16p^2 - 40p + 25$

43. $32a^2 + 48ab + 18b^2$ **44.** $20p^2 - 100pq + 125q^2$

45. $4x^2y^2 + 28xy + 49$ **46.** $9m^2n^2 + 12mn + 4$

47. $(a - 3b)^2 - 6(a - 3b) + 9$ **48.** $(2p + q)^2 - 10(2p + q) + 25$

49. *Concept Check* Match each polynomial in Column I with its factored form in Column II.

I	II
(a) $x^2 + 10xy + 25y^2$	A. $(x + 5y)(x - 5y)$
(b) $x^2 - 10xy + 25y^2$	B. $(x + 5y)^2$
(c) $x^2 - 25y^2$	C. $(x - 5y)^2$
(d) $25y^2 - x^2$	D. $(5y + x)(5y - x)$

50. *Concept Check* Match each polynomial in Column I with its factored form in Column II.

I	II
(a) $8x^3 - 27$	A. $(3 - 2x)(9 + 6x + 4x^2)$
(b) $8x^3 + 27$	B. $(2x - 3)(4x^2 + 6x + 9)$
(c) $27 - 8x^3$	C. $(2x + 3)(4x^2 - 6x + 9)$

Factor each polynomial. ***See Examples 5 and 6.***

51. $9a^2 - 16$

52. $16q^2 - 25$

53. $x^4 - 16$

54. $y^4 - 81$

55. $25s^4 - 9t^2$

56. $36z^2 - 81y^4$

57. $(a + b)^2 - 16$

58. $(p - 2q)^2 - 100$

59. $p^4 - 625$

60. $m^4 - 1296$

61. $8 - a^3$

62. $27 - r^3$

63. $125x^3 - 27$

64. $8m^3 - 27n^3$

65. $27y^9 + 125z^6$

66. $27z^9 + 64y^{12}$

67. $(r + 6)^3 - 216$

68. $(b + 3)^3 - 27$

69. $27 - (m + 2n)^3$

70. $125 - (4a - b)^3$

71. *Concept Check* Which of the following is the correct complete factorization of $x^4 - 1$?

A. $(x^2 - 1)(x^2 + 1)$

B. $(x^2 + 1)(x + 1)(x - 1)$

C. $(x^2 - 1)^2$

D. $(x - 1)^2(x + 1)^2$

72. *Concept Check* Which of the following is the correct factorization of $x^3 + 8$?

A. $(x + 2)^3$

B. $(x + 2)(x^2 + 2x + 4)$

C. $(x + 2)(x^2 - 2x + 4)$

D. $(x + 2)(x^2 - 4x + 4)$

Relating Concepts

For individual or collaborative investigation *(Exercises 73–78)*

The polynomial $x^6 - 1$ can be considered either a difference of squares or a difference of cubes. ***Work Exercises 73–78 in order,*** *to connect the results obtained when two different methods of factoring are used.*

73. Factor $x^6 - 1$ by first factoring as a difference of squares, and then factor further by using the patterns for a sum of cubes and a difference of cubes.

74. Factor $x^6 - 1$ by first factoring as a difference of cubes, and then factor further by using the pattern for a difference of squares.

75. Compare your answers in **Exercises 73 and 74.** Based on these results, what is the factorization of $x^4 + x^2 + 1$?

76. The polynomial $x^4 + x^2 + 1$ cannot be factored using the methods described in this section. However, there is a technique that enables us to factor it, as shown here. Supply the reason why each step is valid.

$$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 \quad \text{\_\_\_\_\_}$$
$$= (x^4 + 2x^2 + 1) - x^2 \quad \text{\_\_\_\_\_}$$
$$= (x^2 + 1)^2 - x^2 \quad \text{\_\_\_\_\_}$$
$$= (x^2 + 1 - x)(x^2 + 1 + x) \quad \text{\_\_\_\_\_}$$
$$= (x^2 - x + 1)(x^2 + x + 1) \quad \text{\_\_\_\_\_}$$

77. Compare your answer in **Exercise 75** with the final line in **Exercise 76.** What do you notice?

78. Factor $x^8 + x^4 + 1$ using the technique outlined in **Exercise 76.**

Factor each polynomial by substitution. ***See Example 7.***

79. $7(3k-1)^2+26(3k-1)-8$

80. $6(4z-3)^2+7(4z-3)-3$

81. $9(a-4)^2+30(a-4)+25$

82. $4(5x+7)^2+12(5x+7)+9$

83. m^4-3m^2-10

84. a^4-2a^2-48

Factor by any method. ***See Examples 1–7.***

85. $4b^2+4bc+c^2-16$

86. $(2y-1)^2-4(2y-1)+4$

87. $x^2+xy-5x-5y$

88. $8r^2-3rs+10s^2$

89. $p^4(m-2n)+q(m-2n)$

90. $36a^2+60a+25$

91. $4z^2+28z+49$

92. $6p^4+7p^2-3$

93. $1000x^3+343y^3$

94. $b^2+8b+16-a^2$

95. $125m^6-216$

96. $q^2+6q+9-p^2$

97. $64+(3x+2)^3$

98. $216p^3+125q^3$

99. $(x+y)^3-(x-y)^3$

100. $100r^2-169s^2$

101. $144z^2+121$

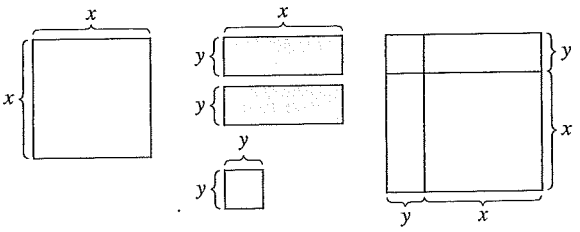
102. $(3a+5)^2-18(3a+5)+81$

103. $(x+y)^2-(x-y)^2$

104. $4z^4-7z^2-15$

105. Are there any conditions under which a sum of squares can be factored? If so, give an example.

106. *Geometric Modeling* Explain how the figures give geometric interpretation to the formula $x^2+2xy+y^2=(x+y)^2$.



Factor each polynomial over the set of rational number coefficients.

107. $49x^2-\frac{1}{25}$

108. $81y^2-\frac{1}{49}$

109. $\frac{25}{9}x^4-9y^2$

110. $\frac{121}{25}y^4-49x^2$

Concept Check Find all values of b or c that will make the polynomial a perfect square trinomial.

111. $4z^2+bz+81$

112. $9p^2+bp+25$

113. $100r^2-60r+c$

114. $49x^2+70x+c$