When a polynomial has a missing term, we allow for that term by inserting a term with a 0 coefficient for it.

EXAMPLE 11 Dividing Polynomials with Missing Terms

Divide $3x^3 - 2x^2 - 150$ by $x^2 - 4$.

SOLUTION Both polynomials have missing first-degree terms. Insert each missing term with a 0 coefficient.

$$
\begin{array}{r l}
 & 3x - 2 \qquad \leftarrow \text{Missing term} \\
x^2 + 0x - 4 & \overline{)\,3x^3 - 2x^2 + 0x - 150} \\
\text{Missing term} & \underline{3x^3 + 0x^2 - 12x} \qquad \text{Insert placeholders for missing terms.} \\
 & -2x^2 + 12x - 150 \\
 & \underline{-2x^2 + 0x + 8} \\
 & 12x - 158 \leftarrow \text{Remainder}
\end{array}
$$

The division process ends when the remainder is 0 or the degree of the remainder is less than that of the divisor. Since $12x - 158$ has lesser degree than the divisor $x^2 - 4$, it is the remainder. Thus, the entire quotient is written as follows.

$$\frac{3x^3 - 2x^2 - 150}{x^2 - 4} = 3x - 2 + \frac{12x - 158}{x^2 - 4}$$

✓ *Now Try Exercise 93.*

R.3 Exercises

Simplify each expression. See Example 1.

1. $(-4x^5)(4x^2)$
2. $(3y^4)(-6y^3)$
3. $n^6 \cdot n^4 \cdot n$
4. $a^8 \cdot a^5 \cdot a$
5. $9^3 \cdot 9^5$
6. $4^2 \cdot 4^8$
7. $(-3m^4)(6m^2)(-4m^5)$
8. $(-8t^3)(2t^6)(-5t^4)$
9. $(5x^2y)(-3x^3y^4)$
10. ***Concept Check*** Decide whether each expression has been simplified correctly. If not, correct it. Assume all variables represent nonzero real numbers.

    (a) $(mn)^2 = mn^2$ **(b)** $y^2 \cdot y^5 = y^7$ **(c)** $\left(\frac{k}{5}\right)^3 = \frac{k^3}{5}$ **(d)** $3^0y = 0$

    (e) $4^5 \cdot 4^2 = 16^7$ **(f)** $(a^2)^3 = a^5$ **(g)** $cd^0 = 1$ **(h)** $(2b)^4 = 8b^4$

Simplify each expression. Assume variables represent nonzero real numbers. See Examples 1–3.

11. $(2^2)^5$
12. $(6^4)^3$
13. $(-6x^2)^3$
14. $(-2x^5)^5$
15. $-(4m^3n^0)^2$
16. $-(2x^0y^4)^3$
17. $\left(\frac{r^8}{s^2}\right)^3$
18. $\left(\frac{p^4}{q}\right)^2$
19. $\left(\frac{-4m^2}{tp^2}\right)^4$
20. $\left(\frac{-5n^4}{r^2}\right)^3$
21. $-\left(\frac{x^3y^5}{z}\right)^0$
22. $-\left(\frac{p^2q^3}{r^3}\right)^0$

Match each expression in Column I with its equivalent in Column II. See ***Example 3.***

I	II	I	II
23. **(a)** 6^0	**A.** 0	**24.** **(a)** $3p^0$	**A.** 0
(b) -6^0	**B.** 1	**(b)** $-3p^0$	**B.** 1
(c) $(-6)^0$	**C.** -1	**(c)** $(3p)^0$	**C.** -1
(d) $-(-6)^0$	**D.** 6	**(d)** $(-3p)^0$	**D.** 3
	E. -6		**E.** -3

25. Explain why $x^2 + x^2$ is not equivalent to x^4.

26. Explain why $(x + y)^2$ is not equivalent to $x^2 + y^2$.

Identify each expression as a polynomial *or* not a polynomial. *For each polynomial, give the degree and identify it as a* monomial, binomial, trinomial, *or* none of these. ***See Example 4.***

27. $-5x^{11}$ **28.** $-4y^5$ **29.** $6x + 3x^4$

30. $-9y + 5y^3$ **31.** $-7z^5 - 2z^3 + 1$ **32.** $-9t^4 + 8t^3 - 7$

33. $15a^2b^3 + 12a^3b^8 - 13b^5 + 12b^6$ **34.** $-16x^5y^7 + 12x^3y^8 - 4xy^9 + 18x^{10}$

35. $\frac{3}{8}x^5 - \frac{1}{x^2} + 9$ **36.** $\frac{2}{3}t^6 + \frac{3}{t^5} + 1$

37. 5 **38.** 9

Find each sum or difference. ***See Example 5.***

39. $(5x^2 - 4x + 7) + (-4x^2 + 3x - 5)$

40. $(3m^3 - 3m^2 + 4) + (-2m^3 - m^2 + 6)$

41. $2(12y^2 - 8y + 6) - 4(3y^2 - 4y + 2)$

42. $3(8p^2 - 5p) - 5(3p^2 - 2p + 4)$

43. $(6m^4 - 3m^2 + m) - (2m^3 + 5m^2 + 4m) + (m^2 - m)$

44. $-(8x^3 + x - 3) + (2x^3 + x^2) - (4x^2 + 3x - 1)$

Find each product. ***See Examples 6–8.***

45. $(4r - 1)(7r + 2)$ **46.** $(5m - 6)(3m + 4)$

47. $x^2\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right)$ **48.** $m^3\left(2m - \frac{1}{4}\right)\left(3m + \frac{1}{2}\right)$

49. $4x^2(3x^3 + 2x^2 - 5x + 1)$ **50.** $2b^3(b^2 - 4b + 3)$

51. $(2z - 1)(-z^2 + 3z - 4)$ **52.** $(3w + 2)(-w^2 + 4w - 3)$

53. $(m - n + k)(m + 2n - 3k)$ **54.** $(r - 3s + t)(2r - s + t)$

55. $(2x + 3)(2x - 3)(4x^2 - 9)$ **56.** $(3y - 5)(3y + 5)(9y^2 - 25)$

57. $(x + 1)(x + 1)(x - 1)(x - 1)$ **58.** $(t + 4)(t + 4)(t - 4)(t - 4)$

Find each product. ***See Examples 8 and 9.***

59. $(2m + 3)(2m - 3)$ **60.** $(8s - 3t)(8s + 3t)$ **61.** $(4x^2 - 5y)(4x^2 + 5y)$

62. $(2m^3 + n)(2m^3 - n)$ **63.** $(4m + 2n)^2$ **64.** $(a - 6b)^2$

65. $(5r - 3t^2)^2$ **66.** $(2z^4 - 3y)^2$

67. $[(2p - 3) + q]^2$ **68.** $[(4y - 1) + z]^2$

69. $[(3q + 5) - p][(3q + 5) + p]$ **70.** $[(9r - s) + 2][(9r - s) - 2]$

71. $[(3a+b)-1]^2$ **72.** $[(2m+7)-n]^2$ **73.** $(y+2)^3$

74. $(z-3)^3$ **75.** $(q-2)^4$ **76.** $(r+3)^4$

Perform the indicated operations. ***See Examples 5–9.***

77. $(p^3-4p^2+p)-(3p^2+2p+7)$ **78.** $(x^4-3x^2+2)-(-2x^4+x^2-3)$

79. $(7m+2n)(7m-2n)$ **80.** $(3p+5)^2$

81. $-3(4q^2-3q+2)+2(-q^2+q-4)$ **82.** $2(3r^2+4r+2)-3(-r^2+4r-5)$

83. $p(4p-6)+2(3p-8)$ **84.** $m(5m-2)+9(5-m)$

85. $-y(y^2-4)+6y^2(2y-3)$ **86.** $-z^3(9-z)+4z(2+3z)$

Perform each division. ***See Examples 10 and 11.***

87. $\dfrac{-4x^7-14x^6+10x^4-14x^2}{-2x^2}$ **88.** $\dfrac{-8r^3s-12r^2s^2+20rs^3}{-4rs}$

89. $\dfrac{4x^3-3x^2+1}{x-2}$ **90.** $\dfrac{3x^3-2x+5}{x-3}$

91. $\dfrac{6m^3+7m^2-4m+2}{3m+2}$ **92.** $\dfrac{10x^3+11x^2-2x+3}{5x+3}$

93. $\dfrac{x^4+5x^2+5x+27}{x^2+3}$ **94.** $\dfrac{k^4-4k^2+2k+5}{k^2+1}$

Relating Concepts

For individual or collaborative investigation *(Exercises 95–98)*

The special products can be used to perform selected multiplications. On the left, we use $(x+y)(x-y)=x^2-y^2$. *On the right,* $(x-y)^2=x^2-2xy+y^2$.

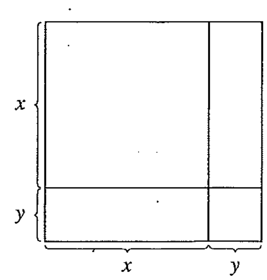
$$\begin{aligned} 51\times 49 &= (50+1)(50-1) \\ &= 50^2-1^2 \\ &= 2500-1 \\ &= 2499 \end{aligned} \qquad \begin{aligned} 47^2 &= (50-3)^2 \\ &= 50^2-2(50)(3)+3^2 \\ &= 2500-300+9 \\ &= 2209 \end{aligned}$$

Use special products to evaluate each expression.

95. 99×101 **96.** 63×57 **97.** 102^2 **98.** 71^2

Solve each problem.

99. *Geometric Modeling* Consider the figure, which is a square divided into two squares and two rectangles.



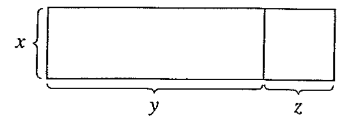
(a) The length of each side of the largest square is $x+y$. Use the formula for the area of a square to write the area of the largest square as a power.

(b) Use the formulas for the area of a square and the area of a rectangle to write the area of the largest square as a trinomial that represents the sum of the areas of the four figures that comprise it.

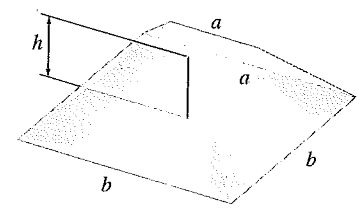
(c) Explain why the expressions in parts (a) and (b) must be equivalent.

(d) What special product formula from this section does this exercise reinforce geometrically?

100. *Geometric Modeling* Use the reasoning process of **Exercise 99** and the accompanying figure to geometrically support the distributive property. Write a short paragraph explaining this process.



101. *Volume of the Great Pyramid* An amazing formula from ancient mathematics was used by the Egyptians to find the volume of the frustum of a square pyramid, as shown in the figure. Its volume is given by



$$V = \frac{1}{3}h(a^2 + ab + b^2),$$

where b is the length of the base, a is the length of the top, and h is the height. (*Source:* Freebury, H. A., *A History of Mathematics,* Macmillan Company, New York.)

(a) When the Great Pyramid in Egypt was partially completed to a height h of 200 ft, b was 756 ft, and a was 314 ft. Calculate its volume at this stage of construction.
(b) Try to visualize the figure if $a = b$. What is the resulting shape? Find its volume.
(c) Let $a = b$ in the Egyptian formula and simplify. Are the results the same?

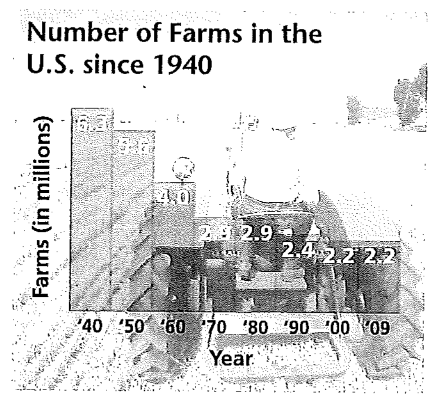
102. *Volume of the Great Pyramid* Refer to the formula and the discussion in **Exercise 101.**

(a) Use $V = \frac{1}{3}h(a^2 + ab + b^2)$ to determine a formula for the volume of a pyramid with square base of length b and height h by letting $a = 0$.
(b) The Great Pyramid in Egypt had a square base of length 756 ft and a height of 481 ft. Find the volume of the Great Pyramid. Compare it with the 273-ft-tall Superdome in New Orleans, which has an approximate volume of 100 million ft^3. (*Source: Guinness Book of World Records.*)
(c) The Superdome covers an area of 13 acres. How many acres does the Great Pyramid cover? (*Hint:* 1 acre = 43,560 ft^2)

(Modeling) Number of Farms in the United States The graph shows the number of farms in the United States since 1940 for selected years. The polynomial

$$0.001147x^2 - 4.5905x + 4595$$

provides a good approximation of the number of farms for these years by substituting the year for x and evaluating the polynomial. For example, if $x = 1960$, *the value of the polynomial is approximately* 3.9, *which differs from the data in the bar graph by only* 0.1.



Source: U.S. Department of Agriculture.

Evaluate the polynomial for each year and then give the difference from the value in the graph.

103. 1940

104. 1970

105. 1990

106. 2009

Concept Check Perform each operation mentally.

107. $(0.25^3)(400^3)$ **108.** $(24^2)(0.5^2)$ **109.** $\frac{4.2^5}{2.1^5}$ **110.** $\frac{15^4}{5^4}$