R.2 Exercises

1. *Concept Check* Match each number from Column I with the letter or letters of the sets of numbers from Column II to which the number belongs. There may be more than one choice, so give all choices.

I		II	
(a) 0	(b) 34	A. Natural numbers	B. Whole numbers
(c) $-\frac{9}{4}$	(d) $\sqrt{36}$	C. Integers	D. Rational numbers
(e) $\sqrt{13}$	(f) 2.16	E. Irrational numbers	F. Real numbers

2. Explain why no answer in **Exercise 1** can contain both D and E as choices.

Concept Check Decide whether each statement is true *or* false. *If it is false, tell why.*

3. Every integer is a whole number.
4. Every natural number is an integer.
5. Every irrational number is an integer.
6. Every integer is a rational number.
7. Every natural number is a whole number.
8. Some rational numbers are irrational.
9. Some rational numbers are whole numbers.
10. Some real numbers are integers.

Let set $A = \left\{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\right\}$. *List all the elements of A that belong to each set.* ***See Example 1.***

11. Natural numbers
12. Whole numbers
13. Integers
14. Rational numbers
15. Irrational numbers
16. Real numbers

Evaluate each expression. ***See Example 2.***

17. -2^4
18. -3^5
19. $(-2)^4$
20. $(-2)^6$
21. $(-3)^5$
22. $(-2)^5$
23. $-2 \cdot 3^4$
24. $-4 \cdot 5^3$

Evaluate each expression. ***See Example 3.***

25. $-2 \cdot 5 + 12 \div 3$
26. $9 \cdot 3 - 16 \div 4$
27. $-4(9-8) + (-7)(2)^3$
28. $6(-5) - (-3)(2)^4$
29. $\left(4 - 2^3\right)\left(-2 + \sqrt{25}\right)$
30. $\left(5 - 3^2\right)\left(\sqrt{16} - 2^3\right)$
31. $\left(-\frac{2}{9} - \frac{1}{4}\right) - \left[-\frac{5}{18} - \left(-\frac{1}{2}\right)\right]$
32. $\left[-\frac{5}{8} - \left(-\frac{2}{5}\right)\right] - \left(\frac{3}{2} - \frac{11}{10}\right)$
33. $\dfrac{-8 + (-4)(-6) \div 12}{4 - (-3)}$
34. $\dfrac{15 \div 5 \cdot 4 \div 6 - 8}{-6 - (-5) - 8 \div 2}$

Evaluate each expression for $p = -4$, $q = 8$, *and* $r = -10$. ***See Example 4.***

35. $-p^2 - 7q + r^2$
36. $-p^2 - 2q + r$
37. $\dfrac{q + r}{q + p}$
38. $\dfrac{p + r}{p + q}$
39. $\dfrac{3q}{r} - \dfrac{5}{p}$
40. $\dfrac{3r}{q} - \dfrac{2}{r}$
41. $\dfrac{5r}{2p - 3r}$
42. $\dfrac{3q}{3p - 2r}$
43. $\dfrac{\frac{q}{2} - \frac{r}{3}}{\frac{3p}{4} + \frac{q}{8}}$
44. $\dfrac{\frac{q}{4} - \frac{r}{5}}{\frac{p}{2} + \frac{q}{2}}$
45. $\dfrac{-(p + 2)^2 - 3r}{2 - q}$
46. $\dfrac{-(q - 6)^2 - 2p}{4 - p}$
47. $\dfrac{3p + 3(4 + p)^3}{r + 8}$
48. $\dfrac{5q + 2(1 + p)^3}{r + 3}$

Identify the property illustrated in each statement. Assume all variables represent real numbers. ***See Examples 5 and 6.***

49. $6 \cdot 12 + 6 \cdot 15 = 6(12 + 15)$

50. $8(m + 4) = 8m + 32$

51. $(t - 6) \cdot \left(\frac{1}{t - 6}\right) = 1$, if $t - 6 \neq 0$

52. $\frac{2 + m}{2 - m} \cdot \frac{2 - m}{2 + m} = 1$, if $m \neq 2$ or -2

53. $(7.5 - y) + 0 = 7.5 - y$

54. $1 \cdot (3x - 7) = 3x - 7$

55. $5(t + 3) = (t + 3) \cdot 5$

56. $-7 + (x + 3) = (x + 3) + (-7)$

57. $(5x)\left(\frac{1}{x}\right) = 5\left(x \cdot \frac{1}{x}\right)$

58. $(38 + 99) + 1 = 38 + (99 + 1)$

59. $5 + \sqrt{3}$ is a real number.

60. 5π is a real number.

61. Is there a commutative property for subtraction? That is, in general, is $a - b$ equal to $b - a$? Support your answer with examples.

62. Is there an associative property for subtraction? That is, does $(a - b) - c$ equal $a - (b - c)$ in general? Support your answer with examples.

Simplify each expression. ***See Examples 5 and 6.***

63. $\frac{10}{11}(22z)$

64. $\left(\frac{3}{4}r\right)(-12)$

65. $(m + 5) + 6$

66. $8 + (a + 7)$

67. $\frac{3}{8}\left(\frac{16}{9}y + \frac{32}{27}z - \frac{40}{9}\right)$

68. $-\frac{1}{4}(20m + 8y - 32z)$

Use the distributive property to rewrite sums as products and products as sums. ***See Example 6.***

69. $8p - 14p$

70. $15x - 10x$

71. $-4(z - y)$

72. $-3(m + n)$

Concept Check Use the distributive property to calculate each value mentally.

73. $72 \cdot 17 + 28 \cdot 17$

74. $32 \cdot 80 + 32 \cdot 20$

75. $123\frac{5}{8} \cdot 1\frac{1}{2} - 23\frac{5}{8} \cdot 1\frac{1}{2}$

76. $17\frac{2}{5} \cdot 14\frac{3}{4} - 17\frac{2}{5} \cdot 4\frac{3}{4}$

Concept Check Decide whether each statement is true *or* false. *If false, correct the statement so it is true.*

77. $|6 - 8| = |6| - |8|$

78. $|(-3)^3| = -|3^3|$

79. $|-5| \cdot |6| = |-5 \cdot 6|$

80. $\frac{|-14|}{|2|} = \left|\frac{-14}{2}\right|$

81. $|a - b| = |a| - |b|$, if $b > a > 0$

82. If a is negative, then $|a| = -a$.

Evaluate each expression. ***See Example 7.***

83. $|-10|$

84. $|-15|$

85. $-\left|\frac{4}{7}\right|$

86. $-\left|\frac{7}{2}\right|$

87. $-|-8|$

88. $-|-12|$

Solve each problem. ***See Example 8.***

89. *Blood Pressure Difference* Calculate the P_d value for a woman whose actual systolic pressure is 116 and whose normal value should be 125.

90. *Systolic Blood Pressure* If a patient's P_d value is 17 and the normal pressure for his gender and age should be 130, what are the two possible values for his systolic blood pressure?

Let $x = -4$ and $y = 2$. Evaluate each expression. ***See Example 9.***

91. $|3x - 2y|$ **92.** $|2x - 5y|$ **93.** $|-3x + 4y|$ **94.** $|-5y + x|$

95. $\dfrac{2|y| - 3|x|}{|xy|}$ **96.** $\dfrac{4|x| + 4|y|}{|x|}$ **97.** $\dfrac{|-8y + x|}{-|x|}$ **98.** $\dfrac{|x| + 2|y|}{-|x|}$

Justify each statement by giving the correct property of absolute value from this section. Assume all variables represent real numbers.

99. $|m| = |-m|$ **100.** $|-k| \geq 0$

101. $|9| \cdot |-6| = |-54|$ **102.** $|k - m| \leq |k| + |-m|$

103. $|12 + 11r| \geq 0$ **104.** $\left|\dfrac{-12}{5}\right| = \dfrac{|-12|}{|5|}$

Find the given distances between points P, Q, R, and S on a number line, with coordinates $-4, -1, 8,$ *and* 12*, respectively.* ***See Example 10.***

105. $d(P, Q)$ **106.** $d(P, R)$ **107.** $d(Q, R)$ **108.** $d(Q, S)$

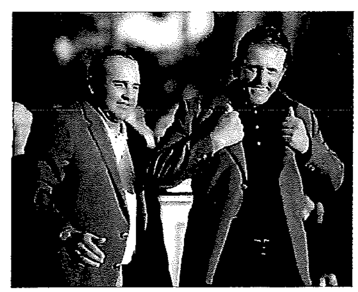
Concept Check Determine what signs on values of x and y would make each statement true. Assume that x and y are not 0. *(You should be able to work these mentally.)*

109. $xy > 0$ **110.** $x^2y > 0$ **111.** $\dfrac{x}{y} < 0$

112. $\dfrac{y^2}{x} < 0$ **113.** $\dfrac{x^3}{y} > 0$ **114.** $-\dfrac{x}{y} > 0$

Solve each problem.

115. *Golf Scores* Phil Mickelson won the 2010 Masters Golf Tournament with a total score that was 16 under par, and Zach Johnson won the 2007 tournament with a total score that was 1 above par. Using -16 to represent 16 below par and $+1$ to represent 1 over par, find the difference between these scores (in either order) and take the absolute value of this difference. What does this final number represent? (*Source:* www.masters.org)



116. *Total Football Yardage* During his 16 years in the NFL, Marcus Allen gained 12,243 yd rushing, 5411 yd receiving, and -6 yd returning fumbles. Find his total yardage (called *all-purpose yards*). Is this the same as the sum of the absolute values of the three categories? Explain. (*Source: The Sports Illustrated Sports Almanac.*)

Blood Alcohol Concentration The blood alcohol concentration (BAC) of a person who has been drinking is approximated by the following expression.

number of oz $\times$ % alcohol $\times$ 0.075 $\div$ body weight in lb $-$ hr of drinking $\times$ 0.015

(*Source:* Lawlor, J., *Auto Math Handbook: Mathematical Calculations, Theory, and Formulas for Automotive Enthusiasts,* HP Books.)

117. Suppose a policeman stops a 190-lb man who, in 2 hr, has ingested four 12-oz beers (48 oz), each having a 3.2% alcohol content. Calculate the man's BAC to the nearest thousandth. Follow the order of operations.

118. Find the BAC to the nearest thousandth for a 135-lb woman who, in 3 hr, has drunk three 12-oz beers (36 oz), each having a 4.0% alcohol content.

119. Calculate the BACs in **Exercises 117 and 118** if each person weighs 25 lb more and the rest of the variables stay the same. How does increased weight affect a person's BAC?

120. Predict how decreased weight would affect the BAC of each person in **Exercises 117 and 118.** Calculate the BACs if each person weighs 25 lb less and the rest of the variables stay the same.

Archie Manning, father of NFL quarterbacks Peyton and Eli, signed this photo for author Hornsby's son, Jack.

Passer Rating for NFL Quarterbacks The current system of rating passers in the National Football League, adopted in 1973, is based on four performance components: completions, touchdowns, yards gained, and interceptions, as percentages of the number of passes attempted. It uses the following formula.

$$\text{Rating} = \frac{\left(250 \cdot \frac{C}{A}\right) + \left(1000 \cdot \frac{T}{A}\right) + \left(12.5 \cdot \frac{Y}{A}\right) + 6.25 - \left(1250 \cdot \frac{I}{A}\right)}{3},$$

where $A =$ attempted passes, $C =$ completed passes, $T =$ touchdown passes, $Y =$ yards gained passing, and $I =$ interceptions.

In addition to the weighting factors appearing in the formula, the four category ratios are limited to nonnegative values with the following maximums.

$$0.775 \text{ for } \frac{C}{A}, \quad 0.11875 \text{ for } \frac{T}{A},$$

$$12.5 \text{ for } \frac{Y}{A}, \quad 0.095 \text{ for } \frac{I}{A}$$

Exercises 121–132 give the 2010 regular season statistics for the top twelve quarterbacks. Use the formula to determine the rating for each. Round each answer to the nearest tenth.

Quarterback, Team	*A* **Att**	*C* **Comp**	*T* **TD**	*Y* **Yds**	*I* **Int**
121. Tom Brady, NE	492	324	36	3900	4
122. Philip Rivers, SD	541	357	30	4710	13
123. Aaron Rodgers, GB	475	312	28	3922	11
124. Michael Vick, PHI	372	233	21	3018	6
125. Ben Roethlisberger, PIT	389	240	17	3200	5
126. Josh Freeman, TB	474	291	25	3451	6
127. Joe Flacco, BAL	489	306	25	3622	10
128. Matt Cassel, KC	450	262	27	3116	7
129. Matt Schaub, HOU	574	365	24	4370	12
130. Peyton Manning, IND	679	450	33	4700	17
131. Matt Ryan, ATL	571	357	28	3705	9
132. Drew Brees, NO	658	448	33	4620	22

Source: www.nfl.com

133. Steve Young, of the San Francisco 49ers, set a full season rating record of 112.8 in 1994 and held that record until Peyton Manning surpassed it in 2004. (As of 2011, Manning's all-time record holds.) If Manning had 336 completions, 49 touchdowns, 10 interceptions, and 4557 yards, for 497 attempts, what was his rating in 2004?

134. Refer to the passer rating formula and determine the highest rating possible (considered a "perfect" passer rating).